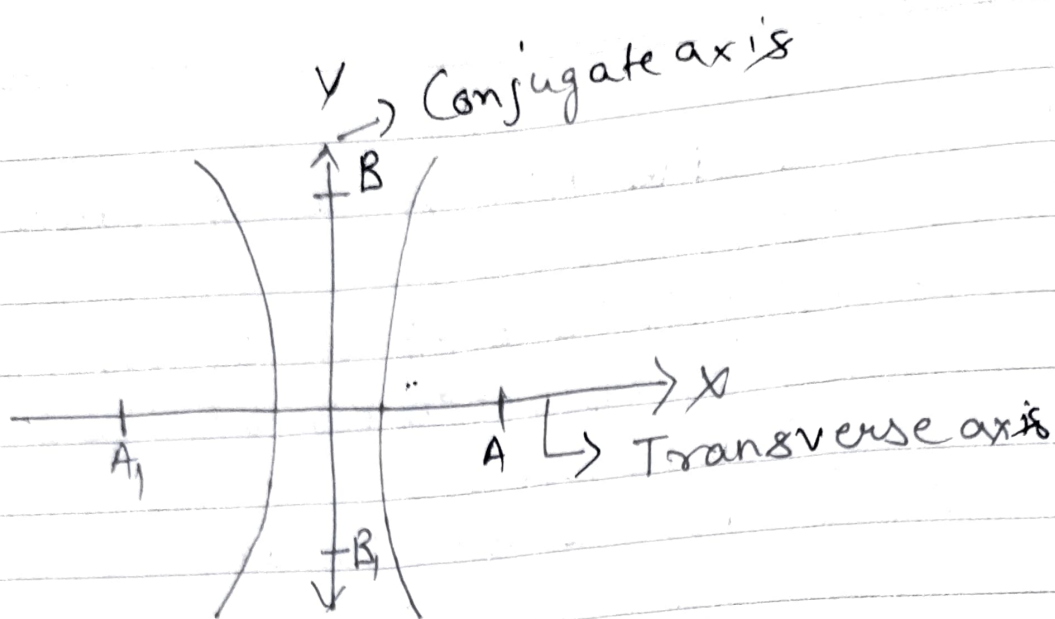
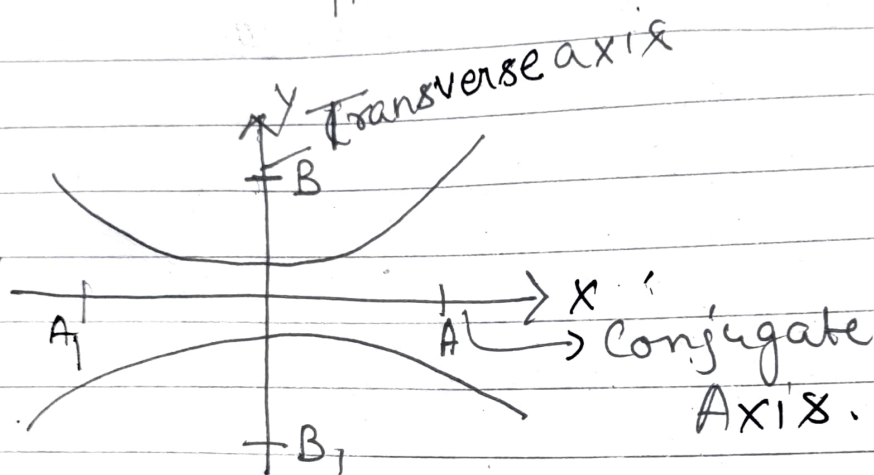


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Normal hyperbola.



Conjugate hyperbola.

The hyperbola which has BB_1 for transverse axis and AA_1 for Conjugate axis is known as Conjugate hyperbola and its equation is given by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Important Points

If eqn of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
then.

1) Eqn of tangent at (x_1, y_1) is.

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

2) Eqn of tangent at $(a \sec \theta, b \tan \theta)$ is

$$\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$$

3) The line $y = mx + \sqrt{a^2 m^2 - b^2}$ touches the hyperbola for all values of m .
where m is the slope.

4) The equation to the normal (x_1, y_1) is

$$\frac{x - x_1}{x_1/a^2} = \frac{y - y_1}{-y_1/a^2}$$

5) The line $lx + my = n$ touches the hyperbola at $a^2 l^2 - b^2 m^2 = n^2$

6) The line $x \cos \alpha + y \sin \alpha = p$ will touch the hyperbola if $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$.

Questions

1) Find the equation of the hyperbola with foci $(6, 4)$ and $(-4, 4)$ and eccentricity $= 2$.

Soln Distance b/w the foci is

$$= \sqrt{(6+4)^2 + (4-4)^2}$$
$$= 10$$

$$\Rightarrow 2ae = 10$$

$$e = 2 \Rightarrow 2 \times a \times 2 = 10$$

$$\Rightarrow a = 5/2 \Rightarrow a^2 = 25/4$$

$$\Rightarrow b = \sqrt{a^2(e^2 - 1)}$$

$$b^2 = a^2(e^2 - 1)$$

$$= \frac{25}{4}(4 - 1) = \frac{75}{4}$$

\therefore Eqn of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{25/4} - \frac{y^2}{75/4} = 1$$

$$\Rightarrow \frac{4x^2}{25} - \frac{4y^2}{75} = 1$$

$$\Rightarrow 12x^2 - 4y^2 = 75$$

Ans

Ques Find the centre & eccentricity of the hyperbola $x^2 - 4y^2 - 2x + 24y - 37 = 0$

Soln Given $x^2 - 4y^2 - 2x + 24y - 37 = 0$

$$\Rightarrow x^2 - 2x - 4y^2 + 24y = 37$$

$$\Rightarrow (x-1)^2 - 1 - 4(y^2 - 6y) = 37$$

$$\Rightarrow (x-1)^2 - 4(y-3)^2 - 9 = 38$$

$$\Rightarrow (x-1)^2 - 4(y-3)^2 = 38 - 36$$

$$\Rightarrow (x-1)^2 - 4(y-3)^2 = 2$$

$$\Rightarrow \frac{(x-1)^2}{2} - \frac{(y-3)^2}{1/2} = 1$$

Put $x-1 = X$, $y-3 = Y$

$$\Rightarrow \frac{X^2}{2} - \frac{Y^2}{1/2} = 1$$

Centre is given if $X=0$, $Y=0$

$$\Rightarrow x=1, y=3$$

Centre is $(1, 3)$

$$\therefore b^2 = a^2(e^2 - 1) \Rightarrow \frac{1}{2} = \frac{2}{4}(e^2 - 1)$$

$$\Rightarrow 4e^2 = \frac{1}{4} + 4 \quad 2e^2 = 2 + \frac{1}{2}$$

$$\Rightarrow 2e^2 = \frac{5}{2} \Rightarrow e^2 = \frac{5}{4} \Rightarrow e = \frac{\sqrt{5}}{2}$$

Ans